Unity Root Matrix Theory
Books 1-5 Errata
Issue 1.0 21/12/2018 R J Miller

This document provides the current, known corrections to all five published Books.

Only algebraic errata or incorrect statements are documented herein. Otherwise, grammar and typographical errors are not corrected except when they affect the understanding of the content.

This document is a working document and is updated as and when new mistakes are found.

Corrections are not provided for the online free papers, albeit such corrections are made directly on occasion, as and when.

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1 Book 1 Physics in Integers

Paper 1 Unity Root Matrix Theory

pg. 11

Paragraph after equation (4.6) beginning,

Lastly, it is noted...

This paragraph is incorrect and matrix **A** is not a projection operator, i.e. it does not project an arbitrary vector on to eigenvector **X**. The usage of symbol **X** here for a vector makes it look like an arbitrary vector, but it is an eigenvector of **A** and therefore, by definition, multiplying **X** by **A** will always return an eigenvalue multiple of **X**, which is not the case for an arbitrary vector. In fact, projection operators do enter URMT later in Section 4 of book IV [4] but most definitely not in the above.

pg. 42

The equations at the foot of the page have an extraneous ‘ $\delta\varepsilon$ ’ symbol in front of them, and their correct layout is as follows, with the ‘ $\delta\varepsilon$ ’ removed from all bar the first line:

$\delta\varepsilon$ terms

(A1.5a) $0 = -x^2 + xyR + xz\overline{Q}$

(4.1a) $x = Ry + \overline{Q}z$.

$\eta\varepsilon$ terms

(A1.5b) $0 = -y^2 + yzP + xy\overline{R}$

(4.1b) $y = \overline{R}x + Pz$.

$\eta\delta$ terms

(A1.5c) $0 = -z^2 + xzQ + yz\overline{P}$

(4.1c) $z = Qx + \overline{P}y$.

Paper 2 Pythagorean Triples as Eigenvectors...

pg. 72

Paragraph after equation (9.16), reference [3] should read reference [4].

pg. 91

Equations (9.9) to (9.11) are missing a transpose symbol on the right-hand side and should read as follows:

(9.9) $$\mathbf{X}^{+} \wedge \mathbf{X}^{0} = -\mathbf{X}^{0} \wedge \mathbf{X}^{+} = \left(\mathbf{X}_{-}\right)^{T}$$
$$\mathbf{X}_{+} \wedge \mathbf{X}_{0} = -\mathbf{X}_{0} \wedge \mathbf{X}_{+} = \left(\mathbf{X}^{-}\right)^{T}$$

(9.10) $$\mathbf{X}^{-} \wedge \mathbf{X}^{+} = -\mathbf{X}^{+} \wedge \mathbf{X}^{-} = 2\left(\mathbf{X}_{0}\right)^{T}$$
$$\mathbf{X}_{-} \wedge \mathbf{X}_{+} = -\mathbf{X}_{+} \wedge \mathbf{X}_{-} = 2\left(\mathbf{X}^{0}\right)^{T}$$

(9.11) $$\mathbf{X}^{0} \wedge \mathbf{X}^{-} = -\mathbf{X}^{-} \wedge \mathbf{X}^{0} = \left(\mathbf{X}_{+}\right)^{T}$$
$$\mathbf{X}_{0} \wedge \mathbf{X}_{-} = -\mathbf{X}_{-} \wedge \mathbf{X}_{0} = \left(\mathbf{X}^{+}\right)^{T} .$$

Paper 3 Geometric and Physical Aspects

pg. 123

Second to last line, the text

radius in the $x - y$ *plane*

should read

radius in the $P-Q$ *plane.*

pg. 128

Line before equation (6.9), the subscript 'm0' on $\delta\mathbf{X}_{m0}$ should be 'm-', i.e. $\delta\mathbf{X}_{m0}$ should be $\delta\mathbf{X}_{m-}$

pg. 150

Equation (12.13)

before
$m >> 0$, $\delta m = 1$
correction
$|m| >> 0$, $|\delta m| = 1$

Paper 4 Solving Unity Root Matrix Theory

pg. 217

Second paragraph, 7th line, text *it would be imply* should read *it would imply*.

pg.219

Paragraph before equation (5.8)

Before
$x^n = -y^n \bmod z$
Corrected
$x^n \equiv -y^n \pmod z$

pg. 242

Equation (B2.1), extraneous superscript

Before
(B2.1) $U_7^3 = \{1,2,4\}$.

corrected

(B2.1) $U_7 = \{1,2,4\}$

The superscript '3' was originally used to denote the exponent to which the root pertains, i.e. cubic here, but this superscript notation was later dispensed with.

pg. 243

Equation (B2.3), see also above,

Before

(B2.3) $U_{13}^3 = \{1,3,9\}$

corrected

(B2.3) $U_{13} = \{1,3,9\}$

Paper 5 Unifying Concepts

pg. 305

Last paragraph, third line from bottom, the text $\mathbf{X}^0 = (\overline{P},\overline{Q},\overline{R})$ should read $\mathbf{X}^0 = (\overline{P},-\overline{Q},\overline{R})$

Paper 6 Non-Unity Eigenvalues

pg. 327

Paragraph after equation (4.2f), first line, the text

the residue is now $\pm C^2$

should read

the residue is now $\pm C^n$

pg. 342

last equation from bottom, un-numbered,

before

$\alpha, \beta, \gamma \to C^2 \alpha C^2, \beta C^2, \gamma C^2$

corrected

$\alpha, \beta, \gamma \to \alpha C^2, \beta C^2, \gamma C^2$

pg. 360

Equation (B7), the sign of *k* should be positive, i.e.

before

$0 = 7^3 + 13^3 - 635^3 - k.7.13.635$

correction

$0 = 7^3 + 13^3 - 635^3 + k.7.13.635$

2 Book 2 Higher Dimensional Extensions

pg. 54

Equations (3.90) and (3.91) are both missing the scalar factor s as in before

$$\mathbf{X}^0 = \begin{pmatrix} 0 \\ \mathbf{X}_3 \end{pmatrix}$$

corrected

$$\mathbf{X}^0 = \begin{pmatrix} 0 \\ s\mathbf{X}_3 \end{pmatrix}$$

In addition equation (3.91) should read, in full,

$$\mathbf{X}^0 \begin{pmatrix} 0 \\ s\mathbf{X}_3 \end{pmatrix} = s\mathbf{X}^0\mathbf{X}_3 = 0$$

For information, the four-element, column vector **X,** referred to in the paragraph above equation (3.19), is first defined in (2.4) p20 but without the scalar factor s.

pg. 175

Second to last paragraph, first line, the text *Pythagorean triple* should read *Pythagorean Quadruple*

pg. 319

Equation (E6), the max range of the zero eigenvector index is $n-3$,

before

$$j = 0 \ldots n-1$$

corrected

$$j = 0 \ldots n-3$$

pg. 339

Definition (I4) An **Excess Dimension**

before

$j = 3 < r < n$

corrected

$j = 3 < r \leq n$

3 Book 3 Mathematical and Physical Advances Volume I

pg. 21

Equations (1.54) and (1.55) omit transposition of vector $\mathbf{X}$

before

(1.54) $\mathbf{X}^{-} = \left(\mathbf{X} \quad -|\mathbf{X}|\right)$
(1.55) $\mathbf{X}^{+} = \left(\mathbf{X} \quad |\mathbf{X}|\right).$

correction

(1.54) $\mathbf{X}^{-} = \left(\mathbf{X}^{T} \quad -|\mathbf{X}|\right)$
(1.55) $\mathbf{X}^{+} = \left(\mathbf{X}^{T} \quad |\mathbf{X}|\right).$

pg. 33

The text

Since $v = c$ (1.108) this is all quite basic as it means that $\mathbf{A}$ is really just the identity matrix i.e.

should read

Since $v = c$ (1.108) this is reduces $\mathbf{A}$ to the following simplistic form:

pg. 126

Last paragraph, first line, the text

In the URM3 SPI, Appendix (J), $\mathbf{X}_0$ *and* $\mathbf{X}_-$ *are treated as velocity and acceleration vectors*

should read

In the dual SPI (4.104), $\mathbf{X}_0$ *and* $\mathbf{X}_-$ *are treated as velocity and acceleration vectors*

pg. 329

Paragraph before Section (10-7), first line, the extraneous superscript of '2' on the equation for ***r*** should be removed.

before

$$r \neq \sqrt{x^2 + y^2 + z^2}^{\,2}$$

correction

$$r \neq \sqrt{x^2 + y^2 + z^2}$$

pg. 330

First equation, top of page, un-numbered, the Pythagorean **T** operator is not equal to the negative inverse.

before

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = -\mathbf{I}_3 = \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

correction

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

pg. 330

Equation (10.81), the Skew **T** operator should be equal to the negative of the identity matrix

before

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

correction

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = -\mathbf{I}_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

pg. 331

Second paragraph, third line, the text '*they form an orthogonal set*' is not true. They do not form an orthogonal set. Whilst the equations (10.87) remain correct, they do not cover the following two inner products, that are non-zero, and hence $\mathbf{X}_{i+}$ is not orthogonal to $\mathbf{X}_{i-}$ and vice-versa:

$$\mathbf{X}_{i+} \cdot \mathbf{X}_{i-} = -2C^2$$

$$\mathbf{X}_{i-} \cdot \mathbf{X}_{i+} = -2C^2$$

Hence the set $\{\mathbf{X}_{i+}, \mathbf{X}_{i0}, \mathbf{X}_{i-}\}$ is not truly orthogonal.

pg. 467

Equation (G3) subscript '+' should be '3+'

Before

$$\frac{d\mathbf{X}_0}{dt_3} = -\mathbf{X}_+$$

correction

$$\frac{d\mathbf{X}_0}{dt_3} = -\mathbf{X}_{3+}$$

4 Book 4 Mathematical and Physical Advances Volume II

pg. 249

Equation (10.24) should read as follows, where the subscript on $\mathbf{X}_{4i+}$ has been changed to $\mathbf{X}_{4i\pm}$, and likewise for $\mathbf{X}_{4j+}$:

(10.24)

$$\mathbf{Q}_{\underline{\mathbf{q}}}\mathbf{X}_{4i\pm} = \pm i\left|\underline{\mathbf{q}}\right|\mathbf{X}_{4i\pm}$$

$$\mathbf{Q}_{\underline{\mathbf{q}}}\mathbf{X}_{4j\pm} = \pm i\left|\underline{\mathbf{q}}\right|\mathbf{X}_{4j\pm}$$

On the same page, 249, second line before equation (10.26), the equation $\lambda_{\underline{\mathbf{q}}} \pm i\left|\underline{\mathbf{q}}\right|$ is missing the equality symbol and should read $\lambda_{\underline{\mathbf{q}}} = \pm i\left|\underline{\mathbf{q}}\right|$.

5 Book 5 A Quark Flavour Model

pg. 20

Equation (1.53), last of the three equations, $\mathbf{J}_0$ should be $\mathbf{J}_x$.

before

$[\mathbf{J}_+, \mathbf{J}_-] = 2\hbar\mathbf{J}_0$

correction

$[\mathbf{J}_+, \mathbf{J}_-] = 2\hbar\mathbf{J}_x$

pg. 25

Second to last paragraph, second line, the text

y and z-axis

should read

y and z axes

pg. 88

Equation (4.13c) is missing an equality sign

before

(4.31c) $\bar{\mathbf{u}}\bar{\mathbf{I}}_+ = \bar{\mathbf{u}}\left(-\mathbf{u}\bar{\mathbf{d}}\right) - \left(\bar{\mathbf{u}}\mathbf{u}\right)\bar{\mathbf{d}} = -\bar{\mathbf{d}}$ using $\bar{\mathbf{u}}\mathbf{u} = 1$ (4.17a)

correction

(4.31c) $\bar{\mathbf{u}}\bar{\mathbf{I}}_+ = \bar{\mathbf{u}}\left(-\mathbf{u}\bar{\mathbf{d}}\right) = -\left(\bar{\mathbf{u}}\mathbf{u}\right)\bar{\mathbf{d}} = -\bar{\mathbf{d}}$ using $\bar{\mathbf{u}}\mathbf{u} = 1$ (4.17a)

pg. 197

First paragraph, last sentence

In fact, every one of the seven matrices $\boldsymbol{\lambda}_i$, $i = 1...7$ *squares to the identity, with a full, cubic, characteristic equation [5] given by*

This is wrong since they do not square exactly to the identity and would be better worded as follows:

In fact, every one of the seven matrices $\boldsymbol{\lambda}_i$, $i = 1...7$ *squares to a diagonal matrix with unity occupying two of the three diagonal positions, i.e. almost like the identity but with one of the diagonal positions zero. Each matrix has a full, cubic, characteristic equation [5] given by*

pg. 202

Top of page, equations (11.17), the identity equation for $\mathbf{I}$ (last equation) is actually only true for the 3x3 Gell-Mann matrices, as covered by URM3, and not URM6 dealt with later in Chapter 13 onward. Whilst this point may be clear given that the entire chapter 11 is titled 'Gell-Mann Matrices', the equations (11.18) are supposed to be valid for SU(N), N>=3, i.e currently URM3 to URM6.

pg. 203

Top of page, equations (11.18), continued from page 202, the second equation involving the identity, i.e. the equation for $\mathbf{X}_{S0}\mathbf{X}^{S0}$, is actually only true for the 3x3 Gell-Mann matrices, see the above correction to page 202 for more details.

pg. 287

Equation (13.80). The bottom right matrix element '$\mathbf{0}^3$' of matrix $\mathbf{I}_3$ should really be written as a 3x3 zero matrix, i.e. $\mathbf{0}_3^3 = \mathbf{0}_3 \otimes \mathbf{0}^3$. Note too that the subscript on matrix symbol $\mathbf{I}_3$ is not the same as the numeric subscript/superscript 3 in $\mathbf{0}_3^3$. $\mathbf{I}_3$ is just a name for the isospin operator for ther third (hence '3') component of isospin, whereas subscript/superscript 3 in $\mathbf{0}_3$ and $\mathbf{0}^3$ respectively, denotes the number of elements in the vector. Neither is $\mathbf{I}_3$ an identity matrix as its use of capital '**I**' might suggest - the **I** here is for **I**sospin.

Appendix (D) pg. 354

Second paragraph, first sentence starting with

The N matrix elements of a Lie group are actually derivable from the $N^2 - 1$ *generator matrices plus the identity matrix*

should read

The matrix elements of a Lie group are actually derivable from the $N^2 - 1$ *generator matrices plus the identity matrix*

And on the the same page, last paragraph, starting with

Now, for a Lie group, if $\mathbf{G}_k$ *is a generator,* $k = 1 \ldots N - 1$

should read

Now, for a Lie group, if $\mathbf{G}_k$ *is a generator,* $k = 1 \ldots N^2 - 1$

Appendix (D) pg. 362

Third paragraph, the sentence starting with

The $N-1$ *diagonal generators commute with all other generators,*

and should read

The $N-1$ *diagonal generators commute with each other,*