Unity Root Matrix Theory
Out with Rest Mass in with Wave-Particle Duality

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Abstract

This short article shows how replacing the rest mass of a particle by a scaled form of velocity leads to a natural wave-particle duality interpretation of the Special Theory of Relativity (STR), and with it a simpler, more symmetric form of the relativistic energy-momentum equation.

The approach is a change of viewpoint or reinterpretation of the existing relativistic energy-momentum equation, brought about by the introduction of a new, reduced velocity M, with a twist at the end as to why M is more than just an ad-hoc convenience to enable the reinterpretation.

Acronyms

DCE : Dynamical Conservation Equation
STR : Special Theory of Relativity
URMT : Unity Root Matrix Theory
URM5 : URMT five dimensions

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The Reduced Velocity

A reduced velocity M is defined by the following Pythagorean relation in terms of the speed of light c and the velocity v:

(1) $c^2 = M^2 + v^2$.

Multiplying (1) by the quantity $(mc)^2$, where m is the relativistic mass, i.e. the measured (observed) mass due to a velocity v, gives

(2) $m^2c^4 = m^2v^2c^2 + m^2M^2c^2$.

By defining the following momentum terms:

(3a) $p_v = mv$, relativistic momentum velocity v

(3b) $p_M = mM$, reduced momentum, reduced velocity M

(3c) $p_c = mc$, photon-equivalent momentum 'mass' m, velocity c,

then (2) becomes

(4) $p_c^2c^2 = p_v^2c^2 + p_M^2c^2$.

Since the first term is the total, relativistic energy squared (E^2), i.e.

(5) $E^2 = p_c^2c^2$, total relativistic energy.

and the second term is the relativistic kinetic energy squared ($p_v^2c^2$), then by comparing (4) with the following energy-momentum equation of STR:

(6) $E^2 = p_v^2c^2 + {E_0}^2$, the relativistic energy-momentum equation

for rest mass energy E_0

(6) $E_0 = m_0c^2$,

then E_0 can be written in terms of the new momentum quantity p_M (3b) as in

(7) ${E_0}^2 = p_M^2c^2$.

Thus, from (7), the reduced momentum p_M effectively replaces a 'rest mass' momentum m_0c, related to it as follows:

(8) $P_M = mM = m_0c \Rightarrow m_0c/m = M$,

which also gives an alternate definition of the reduced velocity M in terms of the speed of light and the relativistic gamma factor γ, i.e.

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(9) $M = \dfrac{c}{\gamma}$, $m/m_0 = \gamma = \dfrac{1}{\sqrt{\left(1-(v/c)^2\right)}}$.

Using (8), the rest mass energy (6) can also now be written as

(10) $E_0 = mMc$.

Superficially then, the momentum equation (4) is just a rewrite of the relativistic energy-momentum equation incorporating a new velocity M, but notice that (4) is a much more symmetric form than (6) in that it is expressed only in terms of velocities M, v and c, and a single mass m, by definition (3) of the momenta p_M, p_v and p_c respectively. Furthermore, (4) is also symmetric upon interchange of p_M and p_v, as is (2) symmetric in velocities M and v, both velocities playing an equal role, with the common mass m cancelling throughout as in (1). This cancelling of m is of note because it means that (1) is now a form of the energy-momentum equation, per unit mass - M effectively playing the role of both m_0 and m.

Wave-Particle Duality

From (5), the total energy E is given by

(11) $E = p_c c$

which is more akin to the familiar, photon (or massless particle e.g. graviton) form:

(12) $E = pc$.

However, dividing (4) by c^2 shows that p_c is really a combined form of momenta for all particles, with or without any rest mass, i.e.

(13) $p_c^2 = p_v^2 + p_M^2$.

If the particle travels at the speed of light then it has zero rest mass, and its energy and momentum is all kinetic:

(14) $v = c \Rightarrow E = p_v c$, $p_c = p_v$, $m_0 = 0$, $M = 0$, massless particle, luminal speed,

and if the particle has a non-zero rest mass and zero speed then its energy is all rest mass and its reduced velocity is now the speed of light:

(15) $v = 0 \Rightarrow E = p_M c$, $p_c = p_v$, $m_0 \neq 0$, $M = c$, stationary particle, non-zero rest mass.

In between zero and the speed of light is the following case of a particle with a non-zero rest mass and non-zero speed

(16) $v > 0 \Rightarrow E = p_c c$, $p_c = \sqrt{p_v^2 + p_M^2}$, $m_0 \neq 0$, $M \neq c$, non-zero rest mass and speed

So, in the case of a massless photon (traditionally thought of as a wave), it has a velocity that of the speed of light $v = c$ and a zero reduced velocity $M = 0$. Conversely, in the case of a stationary particle with a non-zero rest mass, then it has zero velocity $v = 0$, but still has a reduced velocity equal to the speed of light, i.e. $M = c$.

We can therefore think of a rest mass particle as having a speed of light value for M, but zero v, and a massless particle having a speed of light value for v but zero M. In between v and M combine by Pythagoras (1) to the speed of light.

The key point here is that both v and M appear on an equal footing. By the interchange symmetry of (1) with respect to v and M, it matters not whether we consider v or M. We tend to always think in terms of v since it is the velocity we measure upward from zero as we put energy into increasing an object's velocity. But the symmetric form of the velocity equation (1), and momentum equation (13), do not make this distinction. By Pythagoras (1), the velocity magnitude can always be thought of as c, but distributed between M and v. As energy is pumped in the speed v increases, so too the relativistic mass m, but since m cancels in (1), it is just M that is seen to decrease as the speed increases. Conversely, M increases towards c as the speed decreases to zero.

In all three velocity cases, (14), (15) and (16), the energy is written as a momentum form involving the speed of light as in (11) or (12), which is a wave interpretation. But the two cases (15) and (16) are particles with a non-zero mass, classically regarded (pre-quantum mechanics) as non-wave-like objects, for $M > 0$. On the left of (4) therefore is a wave (p_c) and on the right is a particle (p_M, p_v), $M > 0$, and that's how the wave-particle duality interpretation arises.

Of course, this all depends on whether the seemingly arbitrary introduction of the reduced velocity M is acceptable, which may not be the case, and is why this article ends with a twist.

The twist

The earlier introduction of the reduced velocity M (1) might seem like an ad-hoc definition that provides the above re-write and re-interpretation of Einstein's relativistic energy equation - albeit that would be good enough reason in itself since it can lead to a wave-particle interpretation of a classical problem. However, M is a dynamical variable in Unity Root Matrix Theory (URMT), and an element of the 5x5 (URM5), unity root matrix $\mathbf{A}_{50}$ [1], whose characteristic (eigenvalue) equation is the relativistic energy-momentum equation (4) for the single, invariant eigenvalue (big C in URMT), which is directly equated with the speed of light, little c. Furthermore, the URMT potential energy V (per unit mass) is the square of the dynamical variable M, whilst the kinetic energy K (per unit mass), is the square of the velocity v as in:

(17) $V = M^2$, potential energy per unit mass

(18) $K = v^2$, kinetic energy per unit mass

Summing these two terms gives URMT's dynamical conservation equation (the DCE), which is just the total energy, per unit mass

(19) $C^2 = (K + V) = c^2$, total energy per unit mass, the DCE.

Finally, multiplying throughout by mc^2 returns URMT's form of the relativistic momentum equation (4)

(20) $E^2 = (K + V)mc^2 = m^2v^2c^2 + m^2M^2c^2 = p_v^2c^2 + p_M^2c^2 = p_c^2c^2$.

See [2] for the full five-dimensional treatment of URMT and STR..

Summary

URMT embodies rest mass in STR through a dynamical variable, the 'reduced velocity' M, which is also the source of the potential energy term in URM5's energy conservation equation, the DCE, and is the same as the relativistic energy-momentum equation. By introducing M, the concept of a rest mass is effectively dispensed with and, instead, the relativistic mass is used throughout, but now cancels in the energy-momentum equation. The STR energy equations are re-written in a more symmetric form involving the velocities M, v and c, or momenta p_M, p_v and p_c, which leads to a dual wave-particle interpretation.

Lastly, M is a URM5-specific dynamical variable which occupies only the fifth dimension of $\mathbf{A}_{50}$ (top row, left column) [1] - so is M observable and does it really matter, or is it really Matter?

References

[1] The front cover of [2] with an explanation and summary of content, http://www.urmt.org/urmt_mapa_fcv_web.pdf.

[2] Unity Root Matrix Theory, Mathematical and Physical Advances, http://www.authorsonline.co.uk/book/1338/Unity_Root_Matrix_Theory_-_Mathematical_and_Physical_Advances_-_Volume_1/